FAIR US HOUSE REPRESENTATION

Bob Agnew

Chicago ASA Chapter

November 22, 2021

Two Elements based on Decennial Census

- Apportionment of House seats to the states - Accepted Algorithm
  - Constrained rounding problem - 435 integer seats allocated
  - Contentious since 1791 - various methods have been tried
  - Current method (since 1941) can be biased toward smaller states
  - New York and Ohio appear to have been shortchanged this year
- Districting within states - Wild West
  - Subject to brazen gerrymandering
  - Should extract humans from the process
  - Constrained k-means census block group geoclustering works nicely

Apportionment - Divisor Methods

- State with population p has unrounded seat quota 435*p/sum(p) - rounding is unavoidable although larger house size would attenuate impact
- Every state gets at least one House seat
- x is a divisor which is adjusted to get 435 seats - notional population per seat
- d is an increasing divisor function on the nonnegative integers with d(a) in the interval [a,a+1] for every a - focus has been on *means* between a and a+1
- For state with population p, rounded seat allocation:
  - max(1, $\lfloor p/x \rfloor$) if p/x <= d($\lfloor p/x \rfloor$)
  - $\lfloor p/x \rfloor$ + 1 if p/x > d($\lfloor p/x \rfloor$)
- Easy to solve with Excel Solver
- *Non-divisor method due to Hamilton rounds quotas down and awards extra seats to states with highest quota fractional parts - simple and seemingly logical but it generates paradoxes*
- *Definitive reference is Balinski & Young, Fair Representation, Brookings (available at Amazon)*

Apportionment - Comparison of Divisor Methods

Method	Mean	$d(a)$
Adams	Minimum	a
Dean	Harmonic	$a(a+1)/(a+1/2)$
Hill (aka Equal Proportions) *	Geometric	$\sqrt{a(a+1)}$
	Logarithmic	$1/\ln((a+1)/a)$
	Identric	$(a+1)^{a+1}/(ea^a)$
Webster (aka Major Fractions) **	Arithmetic	$a+1/2$
Jefferson	Maximum	$a+1$

* Current Method - mandated by law since 1941

** Normal Rounding - favored by Balinksi & Young

Apportionment - 2021 Results

Hill method is a distinct outlier among reasonable options

Number of States	50	Total Seats	435	Minimum Divisor and Apportionments for Each Divisor Method						
	Apportionment		Minimum	Adams	Dean	Hill	Logarithmic	Identric	Webster	Jefferson
State	Population	Quota	Seats	801,421.2	764,698.9	762,994.4	761,188.6	758,028.8	758,007.5	721,211.8
Montana	1,085,407	1.426	1	2	2	2	1	1	1	1
Nebraska	1,963,333	2.579	1	3	3	3	3	3	3	2
Nevada	3,108,462	4.084	1	4	4	4	4	4	4	4
New Hampshire	1,379,089	1.812	1	2	2	2	2	2	2	1
New Jersey	9,294,493	12.211	1	12	12	12	12	12	12	12
New Mexico	2,120,220	2.785	1	3	3	3	3	3	3	2
New York	20,215,751	26.559	1	26	26	26	27	27	27	28
North Carolina	10,453,948	13.734	1	14	14	14	14	14	14	14
North Dakota	779,702	1.024	1	1	1	1	1	1	1	1
Ohio	11,808,848	15.514	1	15	15	15	16	16	16	16
Oklahoma	3,963,516	5.207	1	5	5	5	5	5	5	5
Oregon	4,241,500	5.572	1	6	6	6	6	6	6	5
Pennsylvania	13,011,844	17.095	1	17	17	17	17	17	17	18
Rhode Island	1,098,163	1.443	1	2	2	2	1	1	1	1

Apportionment - 2011 Results & Links

Webster method is a marginal outlier among reasonable options

Number of States	50	Total Seats	435	Minimum Divisor and Apportionments for Each Divisor Method						
	Apportionment		Minimum	Adams	Dean	Hill	Logarithmic	Identric	Webster	Jefferson
State	Population	Quota	Seats	747,171.1	708,466.2	706,817.0	706,655.3	706,493.7	704,658.3	672,454.0
North Carolina	9,535,483	13.461	1	13	13	13	13	13	14	14
North Dakota	672,591	0.949	1	1	1	1	1	1	1	1
Ohio	11,536,504	16.286	1	16	16	16	16	16	16	17
Oklahoma	3,751,351	5.296	1	6	5	5	5	5	5	5
Oregon	3,831,074	5.408	1	6	5	5	5	5	5	5
Pennsylvania	12,702,379	17.932	1	18	18	18	18	18	18	18
Rhode Island	1,052,567	1.486	1	2	2	2	2	2	1	1

https://www.raagnew.com/us-congressional-apportionments.html

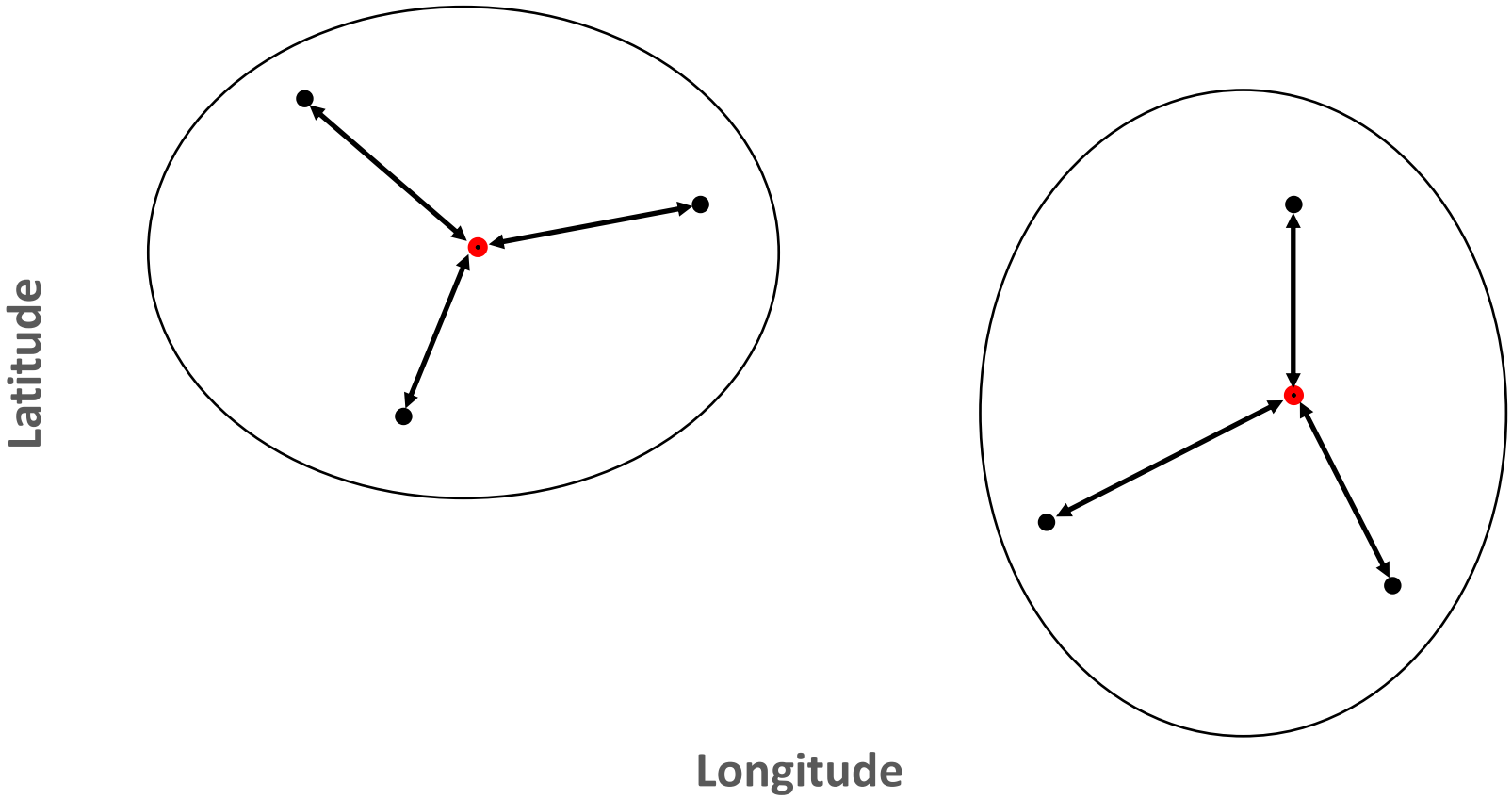
https://github.com/raagnew/CongressionalApportionment

Algorithmic Districting of Census Block Groups

- Constrained k-means clustering puts lower bound on cluster size
- Linear (transportation) program solved at each iteration
- For districting, with varying block group populations, same sized clusters becomes an approximate target
- Census block groups are between larger tracts and tiny blocks
- After discarding zero-population groups, for Illinois:

Block Group Statistic	Population
Range	2 to 7,015
Mean	1,295
Median	1,174

Geoclustering of Census Block Groups



Sum of population-weighted squared distances of block groups to population-weighted cluster centroids is minimized by iteratively adjusting cluster assignments and resulting centroids, subject to the constraint that cluster populations be approximately equal

Districting Links

Research Article on Constrained K-Means Clustering

https://www.microsoft.com/en-us/research/wp-content/uploads/2016/02/tr-2000-65.pdf

https://github.com/raagnew/Constrained-K-Means-Clustering-in-R

https://github.com/raagnew/IdealCongressionalDistricting

https://public.tableau.com/app/profile/bob.agnew