Theoretical Economics Letters, 2024, 14, 2255-2264
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Owner-Union Compensation Game

Robert A. Agnew

Independent Researcher, Palm Coast, Florida, USA
Email: raagnew1@gmail.com

How to cite this paper: Agnew, R. A. (2024). Owner-Union Compensation Game. *Theoretical Economics Letters, 14,* 2255-2264.
https://doi.org/10.4236/tel.2024.146111

Received: August 20, 2024
Accepted: October 12, 2024
Published: November 28, 2024

Abstract

A firm typically consists of an owner and capital provider plus employees who together can create surplus value above individual outside opportunities. We have previously modeled such a firm as a cooperative game across these individual players with specific attention to the core and Shapley value. Here we extend the cooperative game model to include an employee union with a threshold coalition size, including the owner, for contract approval. In this model, with a binding threshold, Shapley value shades more to employees collectively, and in particular to employees with fewer outside opportunities. This seems fair, but Shapley value is not always in the core, and it can therefore be unstable relative to imputations that are shaded more to the owner and higher compensation employees. Shapley value is a standard of fairness, but it is not dominant over core imputations and it can be dominated, and therefore unstable in some cooperative game settings.

Keywords

Compensation, Cooperative Game Theory, Shapley Value, Labor Union, Collective Bargaining

1. Introduction

Suppose that a firm, including its owner and employees, produces surplus value-added above what these individuals can obtain elsewhere. Employees have reserve wages (including benefits) in the labor market. The owner also has a reserve "wage" consisting of an available dividend if he redeploys his capital, as either investor or creditor. Just as his employees have outside opportunities, so does the owner in terms of alternative investments. However, the owner has a special role. He owns the firm and he hires the employees, but he can't obtain anything without at least some employees. Moreover, his current team is exceptional at creating value beyond total reserve wages and he has no guarantee that replacements would do as well. How should surplus value be fairly allocated across the owner and his talented employees? We addressed this question for at-will employees with tools of cooperative game theory in Agnew (2023), with particular focus on the core and Shapley value. Here we extend the cooperative game model to include an employee union with a threshold coalition size, including the owner, for contract approval. As before, our game structure assigns reserve wages to employees in coalitions without the owner but they enable value accumulation in coalitions with the owner, scaled by their reserve wages, *i.e.*, their reserve wages reflect not only their alternate value in the labor market but also drive their incremental value contribution to the owner. However, in the union extension value accumulation is only possible for coalitions exceeding a certain size which enables union contract approval. This size threshold, although normally consisting of the owner and a majority of his employees, is flexible in our model. With a binding threshold above two, Shapley value shades more to employees collectively, and in particular to employees with fewer outside opportunities. This seems fair, but Shapley value is not always in the core, and it can therefore be unstable relative to imputations which are shaded more to the owner and higher compensation employees. There is a large game-theoretic literature on firm vs employees, but most of the literature focuses on non-cooperative models. Sungatullina and Sokolov (2015) portray administration vs employees as a zero-sum game. De Mesnard (2018) seeks a fair compensation distribution between senior managers and middle managers. Wan (2019) and Akinola (2021) independently evaluate game models for pay increases and promotions involving firm and employee strategies for high and low competence employees. Wu (2007) examines both non-cooperative and cooperative models for firm-supported human capital investments by employees. Haber (2006) applies game theory to union negotiations in professional sports, but again his focus is on non-cooperative games like Prisoner's Dilemma and Chicken. In general, the literature on union negotiations is adversarial and non-cooperative. Owen (2001) includes cooperative quotient games with a priori unions, but these are fixed and the unions become distinct unified players. We have chosen a setting with more flexible employee coalitions, but the union still has negotiating power via contract acceptance voting.

Our approach is focused on cooperative compensation bargaining amongst the firm owner and his employees. This follows developments in Stole and Zwiebel (1996a) (SZ) and Brugemann, Gautier, and Menzio (2019) (BGM). SZ developed a sequential (and probabilistic) firm-employee bargaining structure that they claimed led to Shapley value in the corresponding cooperative game. BGM corrected their bargaining structure to achieve that result. They both assumed identical employees although SZ indicated how that assumption could be relaxed. In a tandem publication, Stole and Zwiebel (1996b) examined implications of their findings in organizational design and decision-making for hiring, training, and capital investing. We differentiate employees based on their reserve wages and we don't worry about how bargaining actually occurs, only about what division of firm surplus seems fair. That said, the union extension to our model incorporates an explicit coalition size threshold for collective bargaining. For simplicity and clarity, we assume that all employees are union members, regardless of their outside opportunities. In reality, of course, firms are mixtures of at-will and unionized employees. Our primary game theory reference is Owen (2001). We reviewed requisite elements of cooperative game theory in our previous paper and we assume here that these elements are readily available to the reader. The following section contains a brief review of our previous findings plus the union model extension, including an explicit derivation of Shapley value, followed by a simple illustration and our conclusions.

2. Owner-Union Compensation Game

In Agnew (2023), we defined a cooperative game with a firm owner and capital provider (Player 1) and $n-1$ at-will employees (Players $2,\cdots,n$). In that setting, each individual employee has a reserve annual wage (including benefits) w_i that is available to him in the labor market. Likewise, the owner has a reserve annual dividend w_1 that is available to him if he redeploys his capital elsewhere as an investor or creditor. It is assumed that the current owner and his employees can together create annual value $V > W = \sum_{i=1}^{n} w_i$ but that the owner is indispensable to *any* surplus value creation by his employees. Employees can create surplus, in conjunction with the owner, proportionately to their reserve wages. The owner, on the other hand, can't be readily replaced and he is critical to value creation by his employees. The owner can of course consist of shareholders and their corporate board.

In this context, we define cooperative-game characteristic function v on subsets of $N = \{1,\cdots,n\}$ by $v(S) = \sum_{i\in S} w_i$ if $1 \notin S$ (*i.e.*, employees can't gain anything without the owner) and $v(S) = w_1 + \alpha \sum_{\substack{i\in S \\ i\neq 1}} w_i$ if $1 \in S$ where amplification factor $\alpha = \dfrac{V - w_1}{W - w_1} > 1$ (*i.e.*, surplus value is proportional to included employee reserve wages if the coalition includes the owner, but the owner can't gain anything without at least one employee). It follows in particular that $v(\{i\}) = w_i$ for all $i = 1,\cdots,n$ and that $v(N) = w_1 + \alpha(W - w_1) = V$. Once again, we assume that any current employee can be replaced by the owner at his reserve wage in the labor market but that this replacement doesn't contribute incremental value in the same way as the current incumbent. To simplify notation, we define the set-function $F(S) = \sum_{i\in S} w_i$ for all $S \subset N$ and $F(\phi) = 0$ so that $v(S) = F(S)$ if $1 \notin S$ and $v(S) = w_1 + \alpha F(S - \{1\})$ if $1 \in S$.

Key findings for this cooperative game are:

The game v is convex, which implies that the Shapley value imputation $\phi[v]$ is in the core $C(v)$.

The set of imputations B of the form $x_1 = w_1 + \beta(V - W)$ and $x_i = (\beta + (1-\beta)\alpha) w_i$ for $i > 1$ where $\beta \in [0,1]$ are all in the core $C(v)$, *i.e.*, all degrees of sharing between the owner and his employees are represented in the core.

The Shapley value imputation $\phi[v]$ puts surplus sharing between the owner and his employees at 50% (*i.e.*, $\beta = 1/2$) with $\phi_1[v] = w_1 + (V - W)/2$ and $\phi_i[v] = (1+\alpha)w_i/2$ for $i > 1$. Hence, Shapley value is a particular element of B with equal collective surplus sharing between the owner and his employees. This seems fair but Shapley value is not dominant over other imputations in B.

We now want to extend this game structure to include an employee union requiring a certain voting threshold for contract approval, which must include the owner and a certain number of his employees. This contract is the mutually embraced collective bargaining structure for the firm to create value for the owner and employees beyond their accumulated reserve wages. Ordinarily we might expect the voting threshold to be the owner plus a majority of his union employees, but we will allow the full range of possibilities.

In this context, assuming $n \ge 3$, we define characteristic function u on subsets of N by $u(S) = F(S)$ if $1 \notin S$ or if $1 \in S$ and $|S| < m$, while $u(S) = w_1 + \alpha F(S - \{1\})$ if $1 \in S$ and $|S| \ge m$. Here $|S|$ denotes the size of coalition S and $m \ge 2$ is the contract threshold count, including the owner and a certain number of his employees. When $m = 2$, we have the previous non-union game v in which there is no coalition size threshold and where the owner can deal individually with his employees, *i.e.*, there is a union in name only. When $m = n$, the contract requires 100% approval by all participants, no player has more bargaining power than any other, and not surprisingly we shall see that Shapley value allocates surplus value $V - W$ equally across all participants, owner included. This bargaining structure would obviously be a bitter pill for any owner to swallow. Most interesting cases are of course between these two extremes, but even in this range ($2 < m < n$) instabilities can arise.

Proposition 1. Characteristic function u is superadditive.

Proof. Suppose that $S, T \subset N$ and $S \cap T = \phi$ (*i.e.*, S and T are disjoint subsets) so that $F(S \cup T) = F(S) + F(T)$. If $1 \notin S$ and $1 \notin T$ or if $1 \in S$ and $1 \notin T$ and $|S \cup T| < m$, then

$$u(S \cup T) = F(S \cup T) = F(S) + F(T) = u(S) + u(T).$$

If $1 \in S$ and $1 \notin T$ and $|S| < m$ and $|S \cup T| \ge m$, then

$$u(S \cup T) = w_1 + \alpha F((S \cup T) - \{1\}) = w_1 + \alpha(F(S - \{1\}) + F(T))$$
$$\ge F(S) + F(T) = u(S) + u(T).$$

If $1 \in S$ and $1 \notin T$ and $|S| \ge m$, then

$$u(S \cup T) = w_1 + \alpha F((S \cup T) - \{1\}) = w_1 + \alpha(F(S - \{1\}) + F(T))$$
$$\ge w_1 + \alpha F((S) - \{1\}) + F(T) = u(S) + u(T).$$

Thus $u(S \cup T) \ge u(S) + u(T)$ in all instances where $S \cap T = \phi$ and the game is superadditive. ■

We have proved that u is a legitimate characteristic function. We have not proved that the game u is convex, as the game v is. If the game u were convex, then Shapley value $\phi[u]$ would be in the core $C(u)$ but that is not necessarily true, as we shall see. Interestingly, however, all core imputations for game v are in $C(u)$, including all imputations in *B*. In particular $\phi[v]\in C(u)$, but $\phi[u]$ may not be. If $\phi[u]\notin C(u)$, then it will be unstable, notwithstanding the Shapley connotation of fairness.

Proposition 2. $C(v)\subset C(u)$ so every imputation in the core of game v is in the core of game *u*, including all elements of B and in particular $\phi(v)$.

Proof. $v(S)\ge u(S)$ for all $S\subset N$ by construction. If imputation $x\in C(v)$, then $\sum_{i\in S}x_i\ge v(S)$ for all $S\subset N$ and hence $\sum_{i\in S}x_i\ge u(S)$ as well. It follows that $C(v)\subset C(u)$. ■

We conclude that the core imputations in B have a certain stability for both games v and *u*. We now generalize a result from Agnew (2023).

Proposition 3. If $m=n$, then only the grand coalition has incremental value and all imputations are in the core $C(u)$. If $m<n$, then imputation x is in the core only if $w_1\le x_1\le w_1+V-W$ and $w_i\le x_i\le \alpha w_i$ for $i>1$, *i.e.*, this is a necessary but not sufficient condition for core inclusion.

Proof. If imputation $x\in C(u)$, then $\sum_{i\in S}x_i\ge u(S)$ for all $S\subset N$ and $\sum_{i\in N}x_i=u(N)$. If $m=n$, this is satisfied by any imputation. If $m<n$, we have $x_i\ge w_i$ for all *i*, $x_1=u(N)-\sum_{i\ne 1}x_i\le V-F(N-\{1\})=V-(W-w_1)=w_1+V-W$, and for $i>1$,

$$x_i=u(N)-\sum_{j\ne i}x_j\le V-\left(w_1+\alpha F\left(N-\{i,1\}\right)\right)=\alpha\left(W-w_1\right)-\alpha\left(W-w_1-w_i\right)=\alpha w_i.$$

■

This proposition identifies guardrails for core inclusion. If $m<n$ and $x_i>\alpha w_i$ for any employee $i>1$ in imputation *x*, then $x\notin C(u)$, *i.e.*, employee i is overcompensated. Any such imputation will be unstable and dominated through coalition $S=N-\{i\}$ by imputation y with $y_i=x_i-\varepsilon$ and

$y_j=x_j+\dfrac{\varepsilon}{n-1}>x_j$ for $j\ne i$ and

$\sum_{j\in S}y_j=\sum_{j\in S}x_j+\varepsilon=V-x_i+\varepsilon\le u(S)=V-\alpha w_i$ for sufficiently small ε.

We now explicitly derive the Shapley imputation $\phi[u]$, which for $m>2$ differs from $\phi[v]$.

Proposition 4. Shapley value $\phi[u]$ for game u is defined by

$$\phi_1[u]=w_1+\left(1-\frac{(m-1)(m-2)}{n(n-1)}\right)(V-W)/2 \text{ and for } i>1$$

$$\phi_i[u]=\frac{(m-1)(m-2)}{n(n-1)(n-2)}(V-W)+\left(1+\alpha-\frac{(\alpha-1)(m-1)(m-2)}{(n-1)(n-2)}\right)w_i/2.$$

Proof. For notational simplicity, we denote $z=\phi[u]$. Recall that

$$z_i=\sum_{\substack{S\subset N\\ i\in S}}\gamma(S)\left[u(S)-u\left(S-\{i\}\right)\right] \text{ where } \gamma(S)=\frac{(s-1)!(n-s)!}{n!}=\frac{1}{n\binom{n-1}{s-1}}$$

depends only on $s = |S|$, the size of subset *S*. Now

$$\sum_{\substack{S\subset N\\ 1\in S\\ |S|<m}} \gamma(S)\left[u(S)-u(S-\{1\})\right] = \sum_{\substack{S\subset N\\ 1\in S\\ |S|<m}} \frac{w_1}{n\binom{n-1}{s-1}} = w_1\sum_{s=1}^{m-1}\frac{\binom{n-1}{s-1}}{n\binom{n-1}{s-1}} = \left(\frac{m-1}{n}\right)w_1 \text{ since}$$

there are $\binom{n-1}{s-1}$ ways to select a subset $S\subset N$ containing 1.

$$\sum_{\substack{S\subset N\\ 1\in S\\ |S|\ge m}} \gamma(S)\left[u(S)-u(S-\{1\})\right] = \sum_{\substack{S\subset N\\ 1\in S\\ |S|\ge m}} \frac{w_1+\alpha F(S-\{1\})-F(S-\{1\})}{n\binom{n-1}{s-1}}$$

$$= w_1\sum_{s=m}^{n}\frac{\binom{n-1}{s-1}}{n\binom{n-1}{s-1}} + (\alpha-1)\sum_{s=m}^{n}\frac{\binom{n-2}{s-2}}{n\binom{n-1}{s-1}}\sum_{j\ne 1} w_j$$

$$= \left(\frac{n-m+1}{n}\right)w_1 + (\alpha-1)(W-w_1)\sum_{s=m}^{n}\frac{s-1}{n(n-1)}$$

since there are $\binom{n-1}{s-1}$ ways to select a subset $S\subset N$ containing 1 and there are $\binom{n-2}{s-2}$ ways to select a subset *S* containing 1 and a particular $j\ne 1$. Utilizing the usual integer summation formula $\sum_{j=1}^{k} j = \frac{k(k+1)}{2}$ and adding the two pieces, $z_1 = w_1 + \left(1-\frac{(m-1)(m-2)}{n(n-1)}\right)(V-W)/2$. For $i>1$,

$$\sum_{\substack{S\subset N\\ i\in S\\ 1\notin S}} \gamma(S)\left[u(S)-u(S-\{i\})\right] = \sum_{\substack{S\subset N\\ i\in S\\ 1\notin S}} \frac{w_i}{n\binom{n-1}{s-1}} = w_i\sum_{s=2}^{n}\frac{\binom{n-2}{s-2}}{n\binom{n-1}{s-1}} = w_i/2$$

$$\sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|<m}} \gamma(S)\left[u(S)-u(S-\{i\})\right] = \sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|<m}} \frac{w_i}{n\binom{n-1}{s-1}} = w_i\sum_{s=2}^{m-1}\frac{\binom{n-2}{s-2}}{n\binom{n-1}{s-1}} = \frac{(m-1)(m-2)}{2n(n-1)}w_i$$

$$\sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|=m}} \gamma(S)\left[u(S)-u(S-\{i\})\right] = \sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|=m}} \frac{w_1+\alpha F(S-\{1\})-F(S-\{i\})}{n\binom{n-1}{s-1}}$$

$$= \sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|=m}} \frac{\alpha w_i+(\alpha-1)F(S-\{i,1\})}{n\binom{n-1}{s-1}} = \alpha w_i\frac{\binom{n-2}{m-2}}{n\binom{n-1}{m-1}} + (\alpha-1)\frac{\binom{n-3}{m-3}}{\binom{n-1}{m-1}}\sum_{\substack{j\ne i\\ j\ne 1}} w_j$$

$$= \frac{\alpha(m-1)}{n(n-1)}w_i + \frac{(\alpha-1)(m-1)(m-2)}{n(n-1)(n-2)}(W-w_i-w_1)$$

using the fact that there are $\binom{n-3}{m-3}$ ways to select a subset S of size m containing i and 1 and a particular $j \neq i$ or 1. Finally,

$$\sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|>m}} \gamma(S)\left[u(S)-u(S-\{i\})\right] = \sum_{\substack{S\subset N\\ i\in S\\ 1\in S\\ |S|>m}} \frac{\alpha\left(F(S-\{1\})-F(S-\{i,1\})\right)}{n\binom{n-1}{s-1}}$$

$$= \alpha w_i \sum_{s=m+1}^{n} \frac{\binom{n-2}{s-2}}{\binom{n-1}{s-1}} = \alpha\left(1-\frac{m(m-1)}{n(n-1)}\right) w_i/2.$$

Adding the pieces and simplifying, we get

$z_i = \frac{(m-1)(m-2)}{n(n-1)(n-2)}(V-W) + \left(1+\alpha-\frac{(\alpha-1)(m-1)(m-2)}{(n-1)(n-2)}\right)w_i/2$ for $i>1$. As a check, it can also be verified that $\sum_{i=1}^{n} z_i = V$. ■

When $m=2$, $\phi[u]=\phi[v]$, since there is a union in name only. As m increases above 2, $\phi[u]$ shifts value collectively to the employees. It also shifts value to employees with lesser reserve wages. In the extreme case with $m=n$, everyone, including the owner, gets an identically equal share of the surplus, *i.e.*, $\frac{V-W}{n}$, but again an owner would be unlikely to sign up for this bargaining structure.

3. Hypothetical Example

The numerical entries in Table 1 were created by the author for illustration; they seem reasonable but they obviously don't correspond to any real-world enterprise. We have a steady-state firm whose total value-added, beyond normal expenses, is V = \$2,400,000. Reserve wages total W = \$1,800,000 with surplus $V-W$ = \$600,000 to be allocated across the owner and his employees above reserve wages.

Table 1. Shapley compensation across firm owner and his employees (\$000).

Player	Reserve Wage	Shapley Compensation by Union Contract Approval Threshold			
		2	3	4	5
Owner	\$800	\$1,100	\$1,070	\$1,010	\$920
Empl1	\$400	\$520	\$520	\$520	\$520
Empl2	\$300	\$390	\$395	\$405	\$420
Empl3	\$200	\$260	\$270	\$290	\$320
Empl4	\$100	\$130	\$145	\$175	\$220
Total	\$1800	\$2400	\$2400	\$2400	\$2400
In Core		Yes	Yes	No	Yes

For threshold 2, this is the same as Shapley compensation in **Table 1** of Agnew (2023), but we have added the corresponding columns for thresholds 3 - 5 in this table using the R-function SHAPLEY in the Appendix. We have also checked these Shapley compensations for inclusion in the game core using the general R-function CORE.CHECK in the Appendix. As the contract threshold increases from 2, the owner receives collectively less surplus value above his reserve wage than his employees. Within the employees, the highest reserve wage employee stays the same while the others receive increasing amounts. For thresholds 2 and 3, the Shapley imputation is in the core. For threshold 4, the Shapley imputation is not in the core, a potential source of instability for the Shapley imputation; we could also have verified core exclusion from Proposition 3 since $\$175 > \alpha w_5 = 1.6 \cdot \$100 = \$160$, *i.e.*, this imputation is outside the Proposition 3 guardrails and is easily dominated. For threshold 5, there is no such issue since all imputations are in the core, but 100% contract approval would be difficult to structure in any real contract negotiation, let alone attain. We conclude that threshold 3 provides the most stable binding option in that the Shapley imputation is at least in the core. This case is also intuitive in that it requires agreement by the owner plus 50% of his employees, not an absolute majority but close to it. The threshold 4 option (owner plus 75% of employees) seems to push the envelope too far.

Even where $\phi[u] \in C(u)$, it is not dominant over other core imputations like those in *B*. Having said that, however, Shapley value is widely regarded as a device for fair allocation of surplus value and this union game shows clearly how that allocation shifts with increasing contract thresholds.

4. Conclusion

We have extended our previous owner-employee compensation game to include an employee union with a voting threshold for union contract approval. In this context, with a binding threshold, Shapley value shades more to employees collectively and specifically to employees with fewer outside opportunities.

While this makes sense, Shapley value can be outside the game core, indicating an element of instability. This does not happen for all contract thresholds, only where Shapley appears to award excessive value surplus to lower reserve-wage employees. Shapley value is considered a standard of fair distribution, but it is not dominant over core imputations and in some instances it is clearly dominated by imputations which are shaded more to the owner and higher compensation employees. Shapley value is a standard of fairness, but it is not dominant over core imputations and it can be dominated, and therefore unstable, in some cooperative game settings.

While reasonably comprehensive, our cooperative game model does not cover a firm with a mixture of at-will and unionized employees. It focuses on a steady-state, unionized firm with surplus value-added that is available for sharing among the owner and his talented employees. It therefore ignores unprofitable firms, chaotic firms, and owners with carte-blanche equivalent employee replacement options. The cooperative model also ignores adversarial actions like lockouts and strikes, although they are clearly threats to the bargaining process. Our focus here is on cooperative, good-faith bargaining where both owner and employees create value for a viable firm.

Conflicts of Interest

No potential conflict of interest was reported by the author.

References

Agnew, R. A. (2023). Owner-employee Compensation Game. *Theoretical Economics Letters, 13,* 1632-1638. https://doi.org/10.4236/tel.2023.136093

Akinola, A. T. (2021). Game Theory and Motivation among Enterprises and Employees, a Key to Human Resource Management. *International Journal of Economics and Management Sciences, 10,* 1-4. https://www.hilarispublisher.com/open-access/game-theory-and-motivation-among-enterprises-and-employees-a-key-to-human-resource-management-83193.html

Brugemann, B., Gautier, P., & Menzio, G. (2019). Intra Firm Bargaining and Shapley Values. *The Review of Economic Studies, 86,* 564-592. https://doi.org/10.1093/restud/rdy015

De Mesnard, L. (2018). *Executive Compensation: The Teachings of Game Theory.* SSRN. https://dx.doi.org/10.2139/ssrn.2584600

Haber, L. J. (2006). Labor Negotiations and Game Theory: The Case of Asymmetric Bargaining Power. *Journal of Collective Negotiations (Formerly Journal of Collective Negotiations in the Public Sector), 31,* 21-32. https://doi.org/10.2190/3451-2njm-vge6-en7c

Owen, G. (2001). *Game Theory* (3rd ed.). Academic Press.

Stole, L. A., & Zwiebel, J. (1996a). Intra-firm Bargaining under Non-Binding Contracts. *The Review of Economic Studies, 63,* 375-410. https://doi.org/10.2307/2297888

Stole, L.A. and Zwiebel, J. (1996b) Organizational Design and Technology Choice under Intrafirm Bargaining. *American Economic Review, 86,* 195-222. https://www.jstor.org/stable/2118263

Sungatullina, L. B., & Sokolov, A. Y. (2015). Applying Game Theory to Optimize Expenses for Employees' Remuneration. *Asian Social Science, 11,* 364-368. https://doi.org/10.5539/ass.v11n11p364

Wan, L. (2019). Nash Equilibrium in the Game of Compensation and Promotion between Enterprises and Employees. In *Proceedings of the 1st International Symposium on Economic Development and Management Innovation (EDMI 2019)* (pp. 463-468). Atlantis Press. https://doi.org/10.2991/edmi-19.2019.76

Wu, A. (2007). An Analysis of Employee Investment in Specific Human Capital Based on Game Theory. *Journal of Contemporary Management Issues, 12,* 41-56. https://hrcak.srce.hr/19158

Appendix. R Scripts/Functions

Our analytical functions are delineated within R-scripts. Once those scripts are loaded into an R workspace, either manually or via the source function, they can be run repeatedly for different inputs.

Moreover, the entire workspace can be saved for repeated use.

```
SHAPLEY <- function(x){
# Computes Shapley value for the Owner-Union Compensation Game
V <- x[1] # Annual value to be distributed
m <- x[2] # Voting threshold for union contract, including owner, >= 2
w <- x[-c(1,2)] # Reserve wages, including owner as first element
n <- length(w) # Must be >= 3
W <- sum(w) # Total reserve wages
a <- (V - w[1])/(W - w[1]) # Amplification factor
z <- rep(0,n)
z[1] <- w[1] + .5*(V-W)*(1 - (m-1)*(m-2)/(n*(n-1)))
z[-1] <- (V-W)*(m-1)*(m-2)/(n*(n-1)*(n-2)) + .5*(1+a-(a-1)*(m-1)*(m-2)/((n-1)*(n-2)))*w[-1]
z}

CORE.CHECK <- function(x){
# Checks Shapley value in Owner-Union Compensation Game for core inclusion
V <- x[1] # Annual value to be distributed
m <- x[2] # Voting threshold for union contract, including owner, >= 2
w <- x[-c(1,2)] # Reserve wages, including owner as first element
n <- length(w) # Must be >= 3
W <- sum(w) # Total reserve wages
a <- (V - w[1])/(W - w[1]) # Amplification factor
# Ref code in https://github.com/raagnew/Shapley-Value-in-R
mat <- array(as.integer(intToBits(-1+1:2^n)),dim=c(32,2^n))[1:n,]
mat <- t(mat[,-c(1,2^n)])
VALUE <- function(u){ifelse(all(c(u[1]==1,sum(u)>=m)),w[1]+a*(u[-1]%*%w[-1]),u%*%w)}
value <- NULL
rows <- dim(mat)[1]
for(i in 1:rows){value <- c(value,VALUE(mat[i,]))}
all(mat%*%SHAPLEY(x) >= value)}

> x <- c(2400,3,800,400,300,200,100)
> SHAPLEY(x)
[1] 1070 520 395 270 145
> CORE.CHECK(x)
[1] TRUE
> x[2] <- 4
> SHAPLEY(x)
[1] 1010 520 405 290 175
> CORE.CHECK(x)
[1] FALSE
```